“where children come first”

Little Plumstead CE VA Primary School Calculations Policy

July 2016

What does maths look like at Little Plumstead Primary School?

- To learn and make progress in mathematics, children need to be provided with a rich mixture of language, pictures and experiences to enable them to form their own understanding of the subject. In other words, they need to play with, talk about and see maths in many different ways to help them understand.

- At Little Plumstead, we feel that maths is a subject that should be understood, rather than a series of procedures to be memorised. This is reflected in the planning and the content of maths lessons.

- The calculations policy should show the natural progression that a child should make through their mathematical education, it is not a year by year guide. Children might be working at different stages of the progression in different year groups, this shows that children learn at different speeds and in different ways. This is the advantage of learning maths in a progressive way, as it gives everyone time to make their own understanding of the subject.

- Written methods that are taught should help children to form their own understanding which, in time, will support future mental calculation. Some of the ‘traditional’ calculations strategies that have been taught in schools over the years do not allow children to ‘see’ the maths that they are doing, which means that the children are prone to make mistakes and lose confidence in their mathematical abilities. The methods in this document enable the maths to be seen, so that children can show their thinking, check their calculations and truly understand the maths that they are doing.

- This policy will show **how** we calculate, not **what** we calculate at Little Plumstead.

Mathematical Vocabulary

The use of talk and mathematical language in the classroom is very important in the context of the development of understanding. Children need to be able to read word problems, process the teacher's instructions and discuss mathematical ideas with their teacher and their classmates. These actions, combined with concrete experiences and models and images, will help children to develop their own understanding of the maths that they are learning.

The children will be encouraged to use the following words during maths lessons:

For Equals: the same as, balances with (when comparing amounts), is equal to.

For Addition: add, more, make, sum, total, altogether, one more, two more, ten more…
how many more to make… ? how many more is… than…? Increase, plus, and.

For Subtraction: take (away), leave, how many are left/left over? count back, how many have gone? one less, two less… ten less… how many fewer is… than…? difference between, subtract, fewer, decrease, minus, take from, reduce, take away.

For Multiplication: multiplied by, multiply, product, groups of, times table, times, scale up/down How many times bigger is…? .

For Division: divided by, share, divide, share equally, divisible by, divide into, group, how many groups of … can you take out of…?. X is a tenth of the size of Y, (not ten times smaller)

Misconceptions that some children hold: The word 'sum' can be used in relation to subtraction, multiplication or division i.e 'division sums'. (Of course, sum means add) Equals means ' the answer is'. (This leads to difficulties when solving missing number problems such as 8+4= +5.)

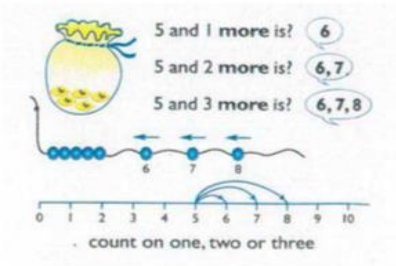
Children's Progression of Understanding

Number Operation	Children's School Year	Phase of the Calculations Policy	Comments
Addition	N, R, 1,	Phase 1	Children in Key Stage 1 need to develop a good understanding of place value and to experience addition in many different forms. Those children who are more confident will start to work in Phase 2 towards the end of Year 1.
Addition	2, 3, 4, 5, 6	Phase 2	Children will develop their understanding of addition as they work within Phase 2. It will be necessary at times for a teacher to draw on the basic concepts in Phase 1 to support a child's understanding.
Subtraction	N, R, 1, 2	Phase 1	Children need to understand the concepts of 'take away' and 'difference'. Through use of objects and comparing lengths to numbers on a number line, children move to more abstract models.
Subtraction	2, 3, 4, 5, 6	Phase 2	Children will show subtraction in various ways with the numberline as the primary model for understanding. Use of objects such as beadstrings and counters will support the children's understanding. Models from Phase 1 will help with this.
Multiplication	N, R, 1, 2	Phase 1	The use of correct language is very important at this stage. As is the use of objects to count in groups and make into small arrays. Links with the numberline should be made through beadstrings.
Multiplication	2, 3, 4, 5, 6	Phase 2	Children build on their understanding from Phase 1 to create more complex arrays and make the link to the grid method. Use of times tables will help children to use the numberline method too. Children without a secure understanding of tables will be taught at Phase 1 to support heir understanding.
Division	N, R, 1, 2	Phase 1	The link with multiplication is key here, as is language too. Children need to do a lot of sharing and grouping of objects to secure their understanding.
Division	2, 3, 4, 5, 6	Phase 2	Once the child has a secure understanding of Phase 1, they can apply their times tables knowledge to Phase 2. Any gaps in their understanding will need to be addressed at Phase 1.

Models and Images for Understanding *Addition* and *Subtraction*

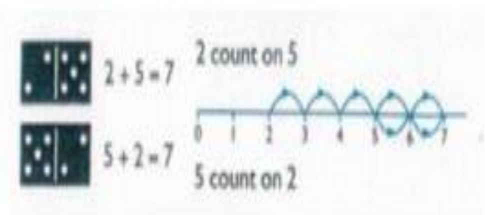
Addition - Phase 1

Combine sets of objects in practical ways. **Count all**, and then **count on**. Make connections with counting up a number track. Lay out objects beneath a number line/ number track to make the connection between the two.



Combine sets of objects:

- Counting all the objects into a pot or bag.
- Count on beads along a bead string.
- Jump up in ones along a number line.



- Start with 2, then count up 5 more to reach the total.
- Or
- Start with 5, then count up 2 to reach the total.
- Line cubes up beneath a number track to show the link between physically adding the cubes and making the jumps up the line.

For each method:

First **count all**; count the first set one-by-one, then continue counting as you count the second set. Then **count on**; remember size of the first set and count on as you add the second set.

Do these calculations on beadstrings to help children form the internal representation of adding beads being similar to counting along numbers on a number track.

Subtraction – Phase 1

It is essential that children are exposed to each of 6 structures of subtraction through their education, with comparison and then taking away being the primary understandings. Children should be taught to compare from Nursery age, so that they gain a sense of the relative size of numbers and amounts. From there they identify the difference between amounts and take away one from the other. These should both be taught at the first point of using subtraction, with a strong emphasis on the language being used.

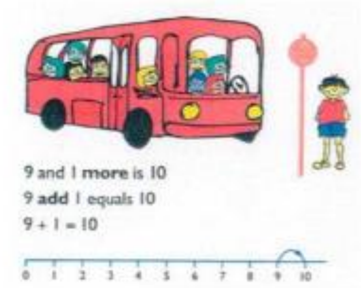
Subtraction can be viewed in 6 structures: (This is not a hierarchy)

- Partitioning and taking away
- Comparison (difference)
- Finding the complement
- Counting back
- The inverse of addition.
- Bridging down through 10.

The hierarchy for the language of comparison:

1. Compare amounts. “There are more red cubes than blue cubes.”
2. Compare numerically. “There are 4 more red cubes than blue.” “There are 16 red cubes but only 12 blue cubes.”
3. Make the link to digits and symbols “16 is 4 more than 12.” **16 – 4 = 12 (**because you have to take 4 away from 16 to make it equal 12.)

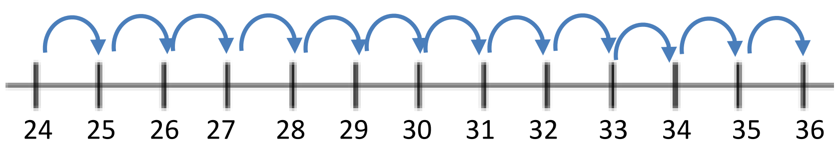
Models and Images for Understanding
Addition and Subtraction



Start with 9 on the bus, one more person gets on board. How many do you have now? Make the explicit link between one more on the bus and one more number along the number line.

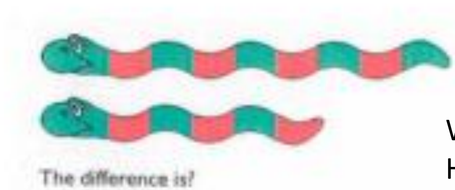
Use this model for adding all single digit numbers. Just increase the number of jumps to fit in with the calculation. This should also be extended to adding a 2-digit number to another 2-digit number.

24+12=

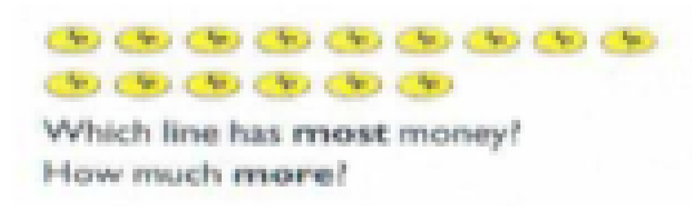


Above each 'jump', the children should write +1 to represent the size of the jump.

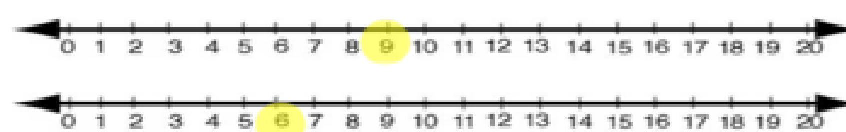
As the children become more secure with the numberline method, they will start to use jumps of differing sizes. The recording of this will need to be adapted to their chosen size of jump.



Which snake is **longer/shorter**?
How much **longer/shorter**?



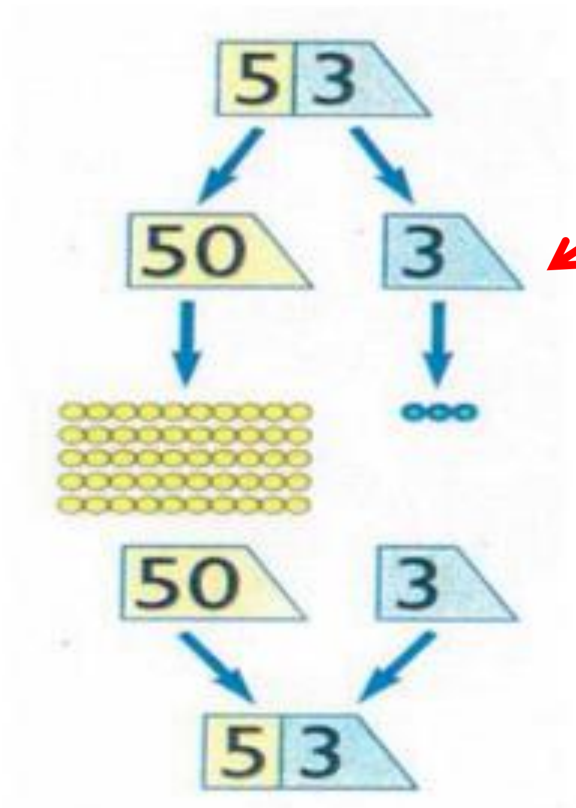
<u>Ask the questions:</u> (For the snakes, coins and number line)
What's the same? What's different?
What do you notice?
Arrange the coins in lines to make the link between the coins lined up and numbers on a number track. 9 is further along the track than 6.



Models and Images for Understanding
Addition and Subtraction

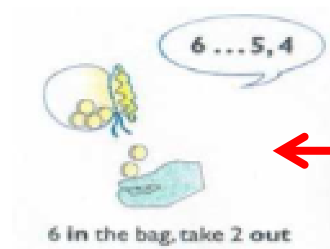
Addition - Phase 2

Partition numbers into tens and ones: 12+23 = (10+2)+(20+3) = (10+20) + (2+3) = 30+5 = 35.
Start with partitioning numbers using Base 10, Cuisenaire Rods or Numicon into tens and ones first, then record as numbers i.e. 53=50+3. Then use this knowledge to represent it on a number line. Finally, do it numerically.

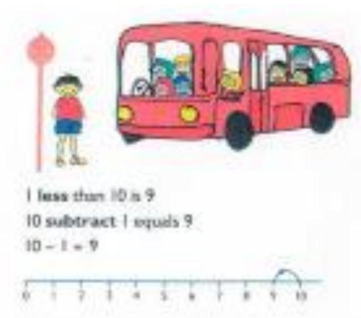


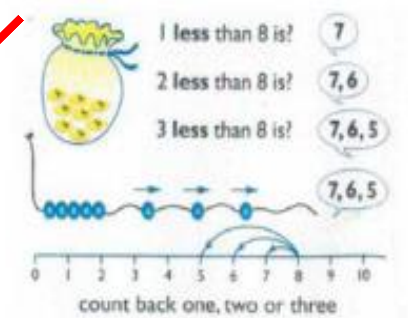
5 tens and 3 left over
(Reinforces the idea of our number system being in base 10.)

Once the children are aware that one number is bigger than another, explore ways of finding the difference between them. Everything that we do at this stage is focussed on helping children to construct their own internal representations of subtraction. These can take many forms, but will be based on one of the following: ***counting out (separating from), counting-back-from, counting-back-to, counting up (difference), inverse of addition or bridging-down-through-ten.*** (Thompson, 2008)



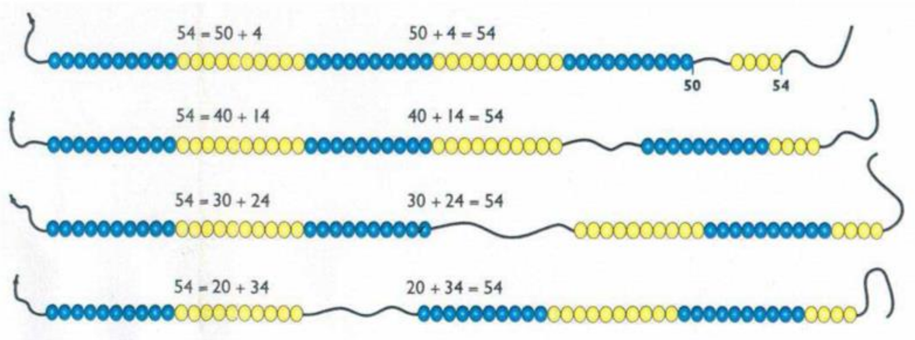
<u>Counting Out (Separating from)</u>
In each case, link the separation from the whole with counting the beads on the bead strings as you take them away. This is then linked to counting back the numbers on a number line.

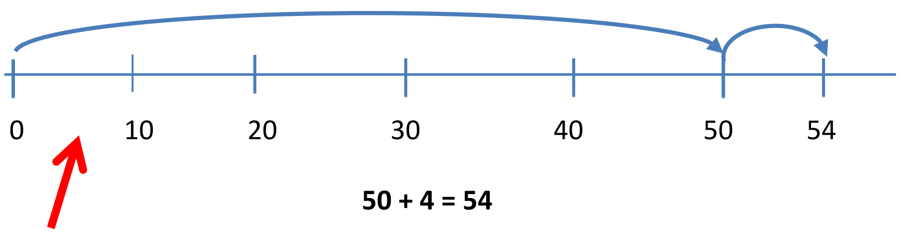




<u>Counting-back-from</u>
3 less than 8 means that you count back three beads. Start with 8 and move three away. At the same time, count back 3 numbers along the number line from 8 to 5.

Models and Images for Understanding Addition and Subtraction





50 + 4 = 54

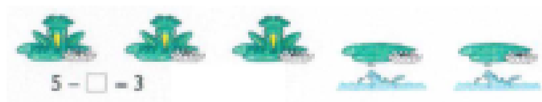
The bead-string representation highlights the structure of the partitioning process. This can also be shown on a number line.

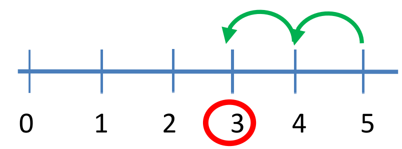
Partitioning on the numberline

Once the children have an understanding of place value and partitioning, you can add by partitioning on the number line. This can be by adding to make the multiple of ten and then add 'lots of ten' until you have added the second amount, or by adding tens to the original number.

Counting-back-to

Start with the first number, identify the target number and count how many beads you need to take away. Link this to identifying the numbers on a number line and counting back to the target number. 5, subtract something equals 3. Count back until you reach 3. How many have you subtracted?



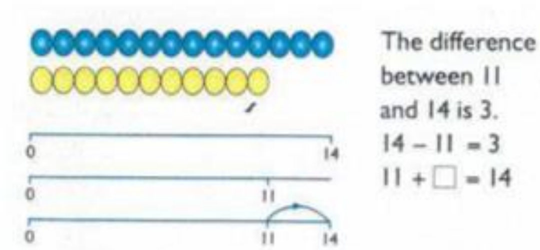


Count back 2 to reach the target of 3.

Counting up (difference) – Step 1

This method exemplifies the gap in the number line between the two numbers of the calculation. Avoid the idea that "you put the small number at one end and the big number at the other". Focus on the relative size of the numbers and identifying the 'difference' between them.

14 – 11 =

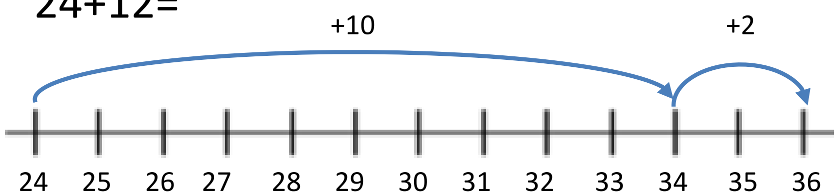


Models and Images for Understanding

Addition and Subtraction

Adding multiples of ten to any number

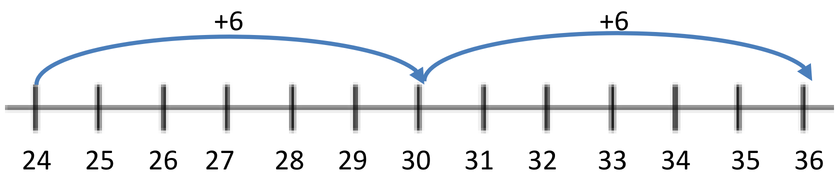
24+12=



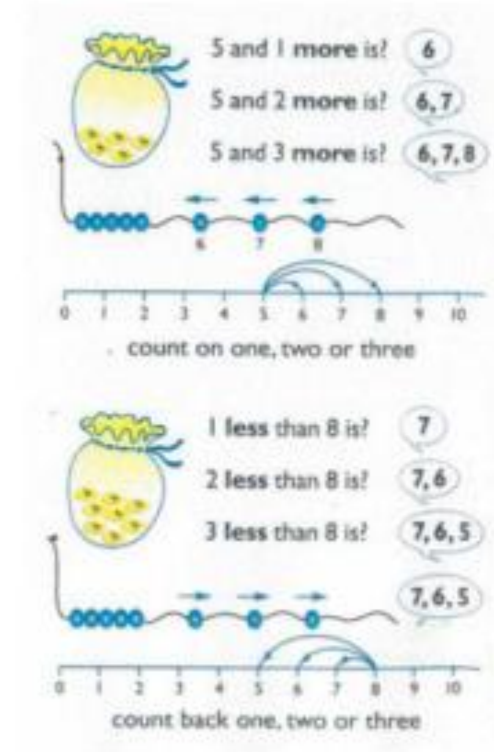
Add the tens, then add the ones. At first, add each ten one at a time, as the children's understanding improves, they can add multiples of ten at a time. This should be extended to hundreds, and decimals too when appropriate.

Bridging through ten with number bonds

24+12=



The next step is to use number bonds to ten in the calculation. In this case, the children add 6 to 24 to make 30, then add on the rest.



The Inverse of addition

This method relies on the children having had opportunities for the development of numerical reasoning. Understanding can be created by encouraging the child to make connections between the written calculation, language and the physical acts of moving a number of beads along the bead string or jumps along the number line.

To help to achieve this, use the following assessment for learning questions and language:

- ***If***, 7 + 4 = 11, what might 11 – 4 = ?
- What do you notice about the number sentences 7+4=11 and 11-4=7 ?
- What's the same and what's different about them?
- Can you think of another number sentence that uses the numbers 11, 4 and 7?
- Can you write down the whole 'calculation family' for 7 + 4 = 11?

Models and Images for Understanding
Addition and Subtraction

<u>Partitioning and recombining . (The 'Bow Tie' method)</u>

34 + 17 =

40 + 11 = 51

125 + 213 =

300 + 30 + 8 = 338

43 + 24 =

40 + 20 = 60

3 + 4 =7

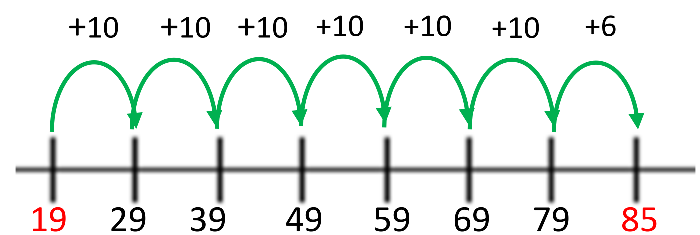
60 + 7 = 67

Subtraction – Phase 2

<u>Difference – step 2</u>

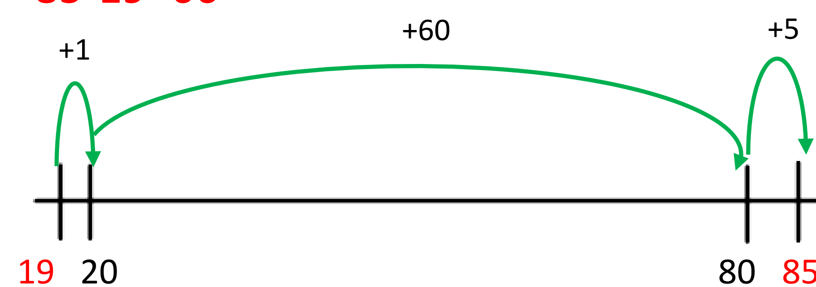
(Adding in 10s)

85-19=66



(Bridging through 10)

85-19=66



The children should be encouraged to draw their number lines with the 'jumps' as representative in size as possible.

This method is then continued to be used with increasingly larger numbers and then decimal numbers.

Models and Images for Understanding
Addition and Subtraction

Children will then be introduced to the informal pencil and paper methods that will build on their existing mental strategies.

This is only to be taught when the children have demonstrated that they have a secure understanding of place value, addition and are consistent when using the numberline strategies.

```
  67                 267
+ 24              +   85
  11 (7 + 4)          12 (7+5)
  80 (60 + 20)       140 (60+80)
  91                 200 (200+0)
                     352
```

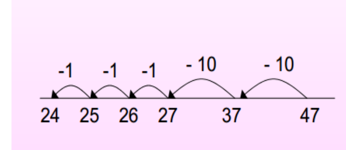
This then progresses to using written methods which will prepare them for the carrying method when it is appropriate.

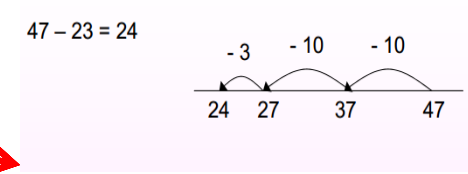
Children are not encouraged to use decomposition for subtraction calculations unless they have shown to have a relational understanding of subtraction and place value. If decomposition is taught without this knowledge, misconceptions can be developed and mistakes made.

All subtraction calculations can be worked out accurately by using the numberline method. The method also supports the development of number sense, which can then be applied to different contexts in written calculation as well as in mental calculation too.

Counting back – Step 2 (using place value

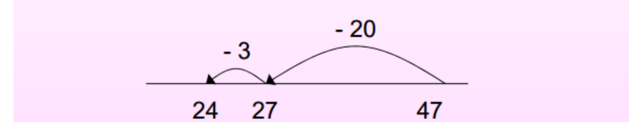
Children begin to use empty number lines (or tracks) to count back. Initially they partition the amount they are subtracting into tens and ones. They then progress to using known number facts to confidently subtract in tens and ones.





They then progress to subtracting the whole group of tens in one jump.

Subtracting the tens in one jump and the units in one jump.



This method is then continued with larger numbers (E.g. HTU) once the children are secure in their understanding of place value. The children should discover that calculating with higher numbers is no harder than doing so with tens and ones, it just has more digits and more steps.

Models and Images for Understanding Multiplication and Division

Four different ways of thinking about multiplication are:

- as repeated addition, for example 3 + 3 + 3 + 3
- as an array, for example four rows of three objects
- as a scaling factor, for example, making a line 3 cm long four times as long.
- as the inverse of division.

Children should experience multiplication in each of these forms during their primary education. Using multiplication in each of these ways in different contexts and in problem solving will help the children to increase their multiplicative fluency and their ability to reason too.

The language of Multiplication:

The use of the multiplication sign can cause difficulties. Strictly, 3 × 4 means four threes or 3 + 3 + 3 + 3. Read correctly, it means 3 multiplied by 4 (*or 3, 4 times*). **However, colloquially it is read as '3 times 4', which is 4 + 4 + 4 or three fours.** Fortunately, multiplication is commutative: 3 × 4 is equal to 4 × 3, so the outcome is the same. It is also a good idea to encourage children to think of any product either way round, as 3 × 4 or as 4 × 3, as this reduces the facts that they need to remember by half.

(From 'Teaching children to calculate mentally' (Department for Education, 2010))

Three different ways of thinking about division are:

- as sharing
- as grouping
- as the inverse of multiplication.

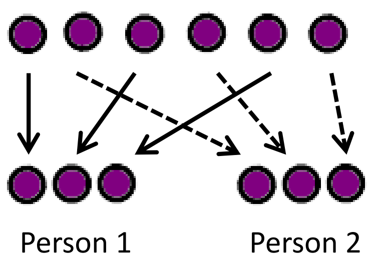
Children should experience division in each of these forms from an early age. Children should be know whether they are sharing or grouping as it is easy to get very muddled. The answer is usually the same, however the conceptual understanding is different.

When discussing multiplication, division should be used as it's inverse, *i.e. 2 x 3 = 6, so how many 2s are there in 6?*

Division – Phase 1

Sharing with objects

6 ÷ 2 = 3



6 objects shared between 2 people. They get 3 objects each.

Sharing is often taught in the earlier years with small numbers and can be calculated easily with objects that can be manipulated. As the numbers get higher, using high numbers of objects can lead to mistakes so other methods need to be used.

Models and Images for Understanding Multiplication and Division

Multiplication – Phase 1

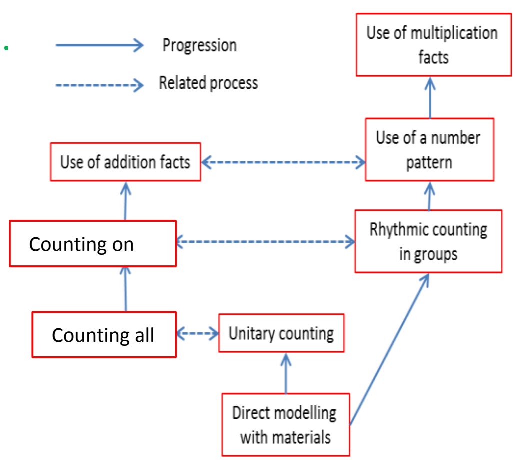
Children's understanding of multiplication starts with unitary counting. (See diagram on next page) using concrete materials. Such as counting cubes; 1, 2, 3, 4…. This of course is strongly linked to addition and the strategy of counting all. Once they can count single objects, they should start to count in twos, fives and tens. Each time, counting groups of objects as they say the number.

Through chanting, seeing the numbers written down and representing the numbers using Numicon or Multilink cubes- in which the patterns in the numbers should be highlighted- the children will create their own internal representations of the multiplication facts.

2, 4, 6, 8, 10, 12, 14, 16, 18…

10, 20, 30, 40, 50, 60, 70…

Patterns:
Numbers end in 2, 4, 6,8, 0.
10, 20, 30 is similar to 1, 2, 3 and each number ends in a 0.

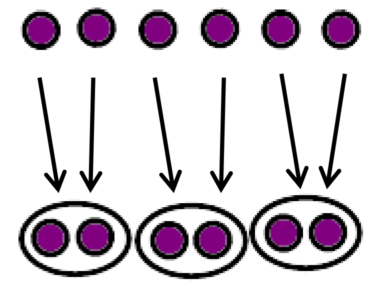


Relationships and progression among addition and multiplication strategies.
(Taken from, Thompson, 2008. pp 118)

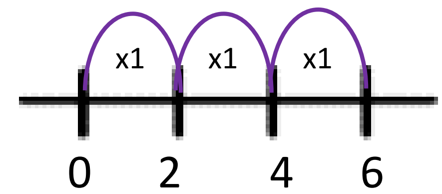
Grouping with objects

6 ÷ 2 = 3

6 divided into groups of 2. There are 3 groups.



How many times does 2 go into 6?
How many 2s are there in 6?



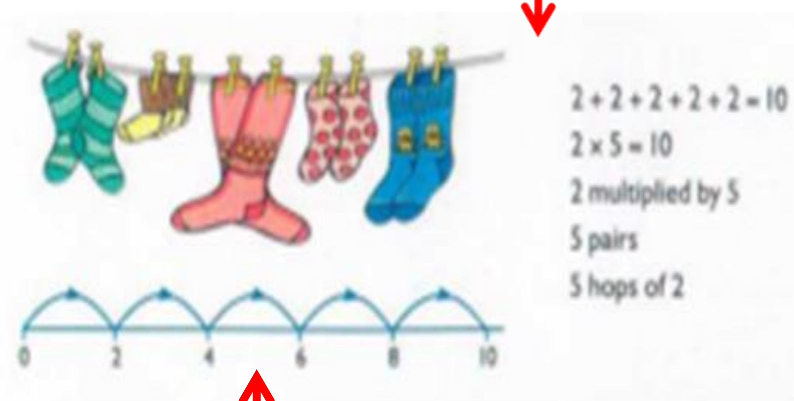
Line the objects up next to the numberline so that the children can see the connection between the line of objects and the numbers on the numberline.

A strong link should be made between division on a numberline and multiplication on a numberline, as they are essentially the same. This also reinforces the concept of the inverse.

Models and Images for Understanding Multiplication and Division

Repeated addition

This model shows that the repeated addition structure of multiplication can be easily represented on a number line. As children count up in 2s, they can count up the jumps on the number line. Initially, this should be represented on a number track which displays each number from 1 to 20.



Equal Groups

As part of helping the children to develop the concept of repeated addition, use the language of multiplication carefully.
For the example above; 2+2+2+2+2=10 which is the same as 2x5=10, the language to use to aid conceptual development is: **2, 5 times.**
Using the phrases '**5 lots of 2**' or '**2 lots of 5**' can be confusing. If you made these as piles of cubes they would look very different, despite ultimately showing 10 cubes in total. This concept is called commutativity. a x b = b x a

This can also be calculated on a numberline.

Models of Multiplication and Division – The Inverses

All of the models of multiplication can and should be used to develop children's understanding of division as the inverse of multiplication.

Using these models should be employed where practical, for instance making or drawing arrays to illustrate the calculations 6÷2 or 20÷5 is much easier than doing the same thing for 250÷10.

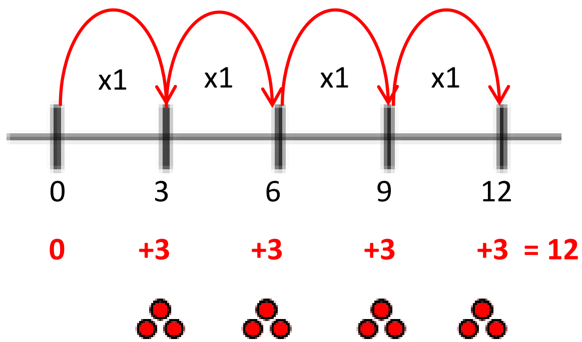
For higher numbers, using informal jottings and related number facts is a quick and efficient way of dividing by using what you know about multiplication.

The number line method for division looks very similar to how it does for multiplication. The key difference is the starting point. For the calculation 96÷4, you are working out how many 4s there are in 96 by starting at 0 and seeing how many 4s there are until you get to 96. The related multiplication would be 4x24=96, in which you are multiplying 4 by 24 to find the answer.

Models and Images for Understanding
Multiplication and Division

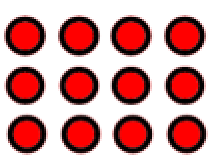
Repeated addition / equal groups - continued

3 x 4 = 12 *(3, 4 times)*



Arrays

3 x 4 = 12 *(3, 4 times)*



Showing multiplication as an array in Phase 1 is essential to help children see that multiplication is commutative and to enable them to really understand how the grid method works in Phase 2.

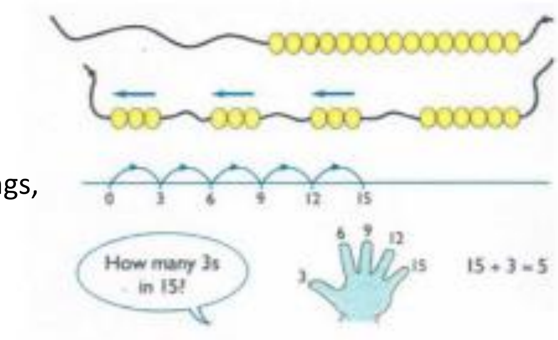
This array could show 3x4 <u>or</u> 4x3. It doesn't matter whether you read it as a column of 3 dots, 4 times, or a row of 4 dots, 3 times. What is important is that the children see the commutativity.

Division – Phase 2

Division on a numberline

15 ÷ 3 = 5

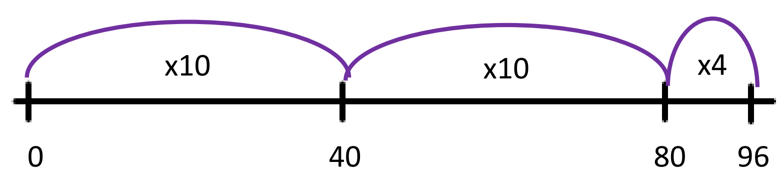
Use other representations, such as bead strings, to support the understanding of division on the number line.



96 ÷ 4 = 24

Division Fact Box
List multiplication facts that can be used in the calculation.

Fact Box
4x10=40
4x20=80
4x5=20
4x2=8
4x4=16

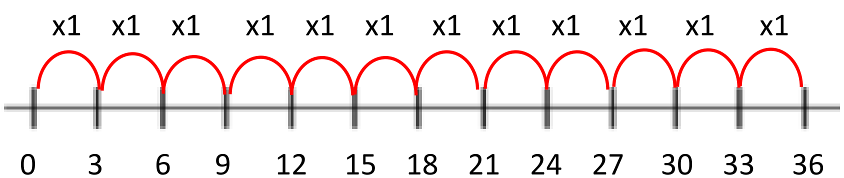


Models and Images for Understanding
Multiplication and Division

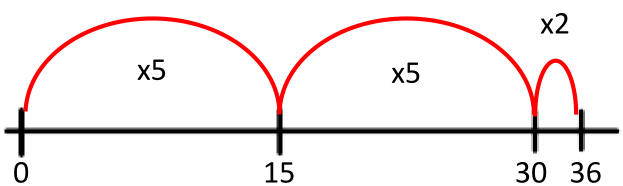
Multiplication – Phase 2

Repeated addition / equal groups – Phase 2

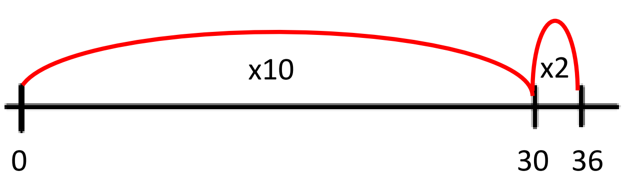
3 x 12 = 36 *(3, 12 times)*



Move from the repeated addition/equal groups model in Phase 1, to using known number facts such as 3x2 or 3x5 to create fewer jumps along the number line, which will also reduce the chance of making mistakes in calculation.



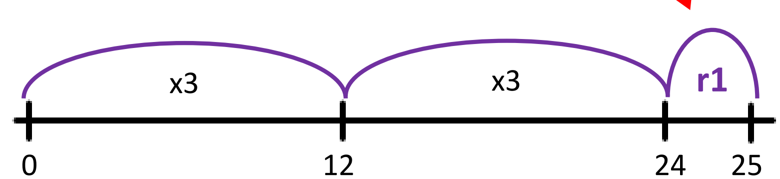
This can then be extended to using multiples of 10 to make the method more efficient.



Division with remainders

25 ÷ 4 = 6 r1

Show the remainder in a different colour.



Numberline vesus more formal methods.

This model of division on a numberline can be extended for any combination of numbers, regardless of their magnitude. This includes using decimals and fractions. The open nature of the model allows the mathematician to apply their own values to the line and to the size of each jump.

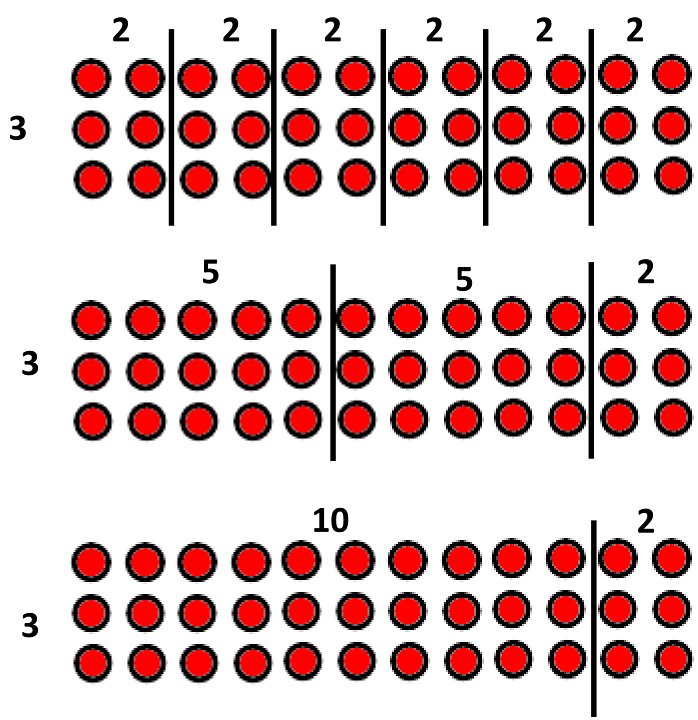
The 'bus stop' method of division has been shown (by the Norfolk Calculations Research) to be confusing and to have a low success rate compared to the numberline method.

Models and Images for Understanding *Multiplication* and *Problem Solving*

Arrays → The Grid Method – Part 1

Once children are able to represent multiplication as an array, they can start to divide up each of the numbers in the calculation to make finding the total easier. Using multiples of 10 is a preferred method, but it is not the only way. Children should be encouraged to see numbers as totals of more than one array.

Different ways of calculating **3 x 12:**

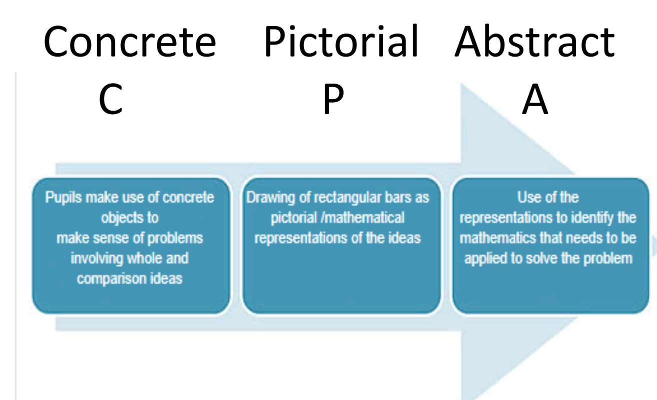


Problem Solving Approaches

Problem solving can take many forms, one of which is word problems. To aid the solving of word problems, many approaches have been used by teachers, including the RUCSAC neumonic (**R**ead **U**nderline **C**alculation? **S**olve **A**nswer **C**heck). A more visual approach is the 'Singapore Bar' or 'Bar Method'.

The Bar Method

The Bar Method is not a calculating tool, rather it is a representation that shows the structure of a word problem. By revealing the structure, it is easier to **see** which parts of the problem are **known** and which parts are **unknown**. From this point, you can select the number operation(s) that you require and can solve the problem. This follows the Concrete-Pictorial-Abstract (CPA) model of conceptual development.



Models and Images for Understanding Multiplication and Problem Solving

Arrays → The Grid Method – Part 2

Once children have experienced different ways of splitting up arrays, they should start to turn this into the conventional layout of the grid method for multiplication. It is important that the children are aware that the different rectangular sections are not the same size as each other. This understanding will be developed through the manipulation of arrays in part 1.

Before calculating, the children should **make an estimate** of the anticipated answer. Adding up the numbers in the boxes should be done in the most efficient way. This may be different for each child. A preferred way is to find he total of the rows and then perform a vertical calculation on the right-hand-side of the grid.

12 x 35 = 420 *Estimate: >350 (Because 10 x 35 = 350)*

x	30	5
10	300	50
2	60	10

300 + 50 + 60 + 10 = 420

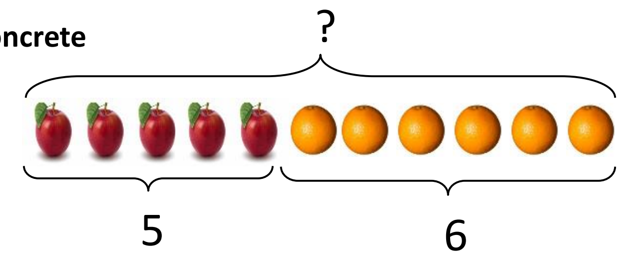
OR: **300 + 50 = 350**
60 + 10 = 70

300
120
420

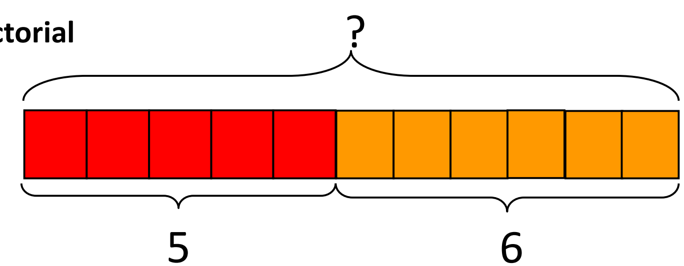
Part-whole model for addition and subtraction

There are 5 apples and 6 oranges. How many pieces of fruit altogether?

Concrete



Pictorial



Abstract

Apples + Oranges = Total fruit. 5+6=?
If the total was known but the oranges were not, then it would be: total fruit – apples = oranges.

Models and Images for Understanding Multiplication and Problem Solving

Long Multiplication?

The 2014 National curriculum makes many references to ‘Formal methods of calculation’ and ‘Long Multiplication’.

The Norfolk Calculations Research showed that children experience a high level of success when using physical representations and jottings when they are using multiplication. Children should be taught to create different arrays and make the link between the objects that they have on their tables and the numberline that they are drawing. This in turn should help them to form a more secure understanding of the concept of multiplication and then would not have to reply on a digit based algorithm such as long multiplication.

It is more important for children to be able to calculate accurately than to be able to follow an algorithm accurately.

Marks in the Key Stage 2 SAT papers will be awarded for correct answers. There will be marks for use of formal methods, but only where the written answer is incorrect. If the answer is correct, then full marks will be awarded.

The bar method can also be used to help solve problems relating to multiplication, division, fractions, ratio and proportion. In each case, the user needs to consider which quantities or relationships they know, and which quantities or relationships they don’t know. Through representing each part with bars, they can then deduce the parts unknown and solve the problem.

In each case, the process should start with the concrete model before moving onto a pictorial representation and then finally by using an abstract representation in the form of a bar, or bars.

Calculations Policy - Appendix

Progression of Understanding in Formal Methods of Calculation

Information and Guidance on the Teaching of Formal Methods of Calculation.

The thinking behind this appendix can be best described by the following extracts taken from *Understanding Mathematics for Young Children.* Haylock and Cockburn (2013). Pg. 185:

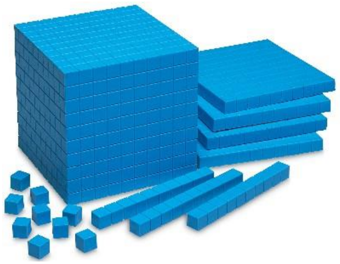
> ***'…there is no case for introducing children to vertical layouts before they have developed a basic understanding of place value.'***
>
> ***'Early introduction of vertical layout can be positively harmful.'***
>
> ***'Mental strategies based on visual or pictorial images, such as movements on the hundred square or along an empty number line, should be the major priority for the development of confidence and success in handling numbers, definitely for children under the age of 9.'***

With these considerations in mind, the teaching of formal methods of calculation should **only** be taught under the following circumstances:

- The child has shown a secure understanding of informal methods of calculation.
- The child has a secure understanding of their times tables up to 10x10.
- The methods are taught as an alternative to informal methods, not as the default method.
- If a child becomes confused by the formal algorithms, then revert back to the progression within the calculations policy to ensure that they are confident in calculation.

Formal Algorithms for…**Addition**

The important considerations when teaching the formal algorithms for addition centre on the understanding of place value. To ensure that understanding is not lost to a simple process of subtracting the digits that are sitting on top of each other. To scaffold this understanding, teachers should plan to use concrete representations such as ‘Base 10 / Dienes blocks’ or money (1ps, 10ps and £1s).



This represents a step on from the partitioning method of addition as it is the first time that children will see a vertical layout.

For calculating 372 + 247 =

This stage should not be rushed as children need to comprehend that…

372 + 247 = 300 + 200 + 70 + 40 + 2 + 7 = 500 + 110 + 9 = 619

<u>Stage 1:</u>

300 + 70 + 2
200 + 40 + 7

500 + 110 + 9 = 619

At this stage, the children will be inclined to add the hundreds first, then the tens before adding ones. As this will have been the order of calculation when they have added by partitioning. When it is appropriate, the teacher should move them towards adding the ones first, so that they are prepared for stages 2 and 3.

Formal Algorithms for...Addition

Stage 2:

```
   3 7 2
 + 2 4 7
 -------
       9   (2+7)
   1 1 0   (70 + 40)
   5 0 0   (300 + 200)
 -------
   6 1 9
```

Stage 2 builds on the understanding that has been maintained in **stage 1**. This is the first time that the algorithm turns from a partitioning based calculation to one that could be 'digit based' . It is very easy to forget that in the stage 2 calculation , you do not calculate 7 + 4, you calculate 70 + 40. This is why it is important to write calculation in the brackets next to the partial sums.

The next developmental step is to do the same process, but without the calculations in the brackets. If the children make conceptual errors, then they should be taken back to **stage 1**.

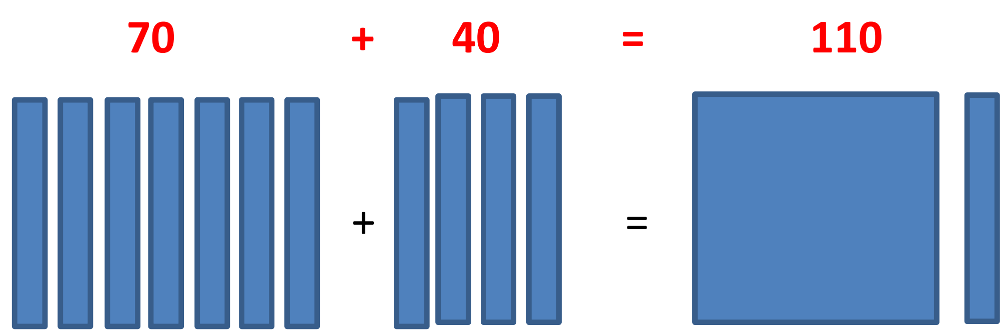
The next step is to remove the bracketed annotations as the calculation becomes more abstract.

Stage 3:

```
   3 7 2
 + 2 4 7
 -------
   6 1 9
 -------
   1
```

The conceptual jump that children need to make is in the 'carrying' of 10 from one column to the next. This is best illustrated by using 'Base 10' resources (below).

Using Base 10



Formal Algorithms for…Subtraction

Subtraction is the number operation in which children tend to make the most ‘little mistakes’ in their calculation. Decomposition as a method (**stage 3**) can be taught for understanding, however it is often taught as a set of procedures to reach an answer.
By following **stages 1 to 3**, and by explaining the reasoning behind each stage, you can ensure that the children understand what they are calculating when they get to **stage 3**. The key point of confusion comes when children are required to ‘borrow 10’ from the column to the left. This needs to be explained by following the concrete-pictorial-abstract model to help the children develop conceptual understanding.

For calculating 448 – 267 =

Stage 1:

$$\begin{array}{r} 400 + 40 + 8 \\ -\ 200 + 60 + 7 \\ \hline 200 - 20 + 1 \quad = 181 \\ \hline \end{array}$$

As with stage 1 of addition, the children are encouraged to see the calculation as a series of subtractions following the partitioning of the number into hundreds, tens and units.
The potential misconception comes when the children are asked to perform the calculation 40 – 60. The temptation will be to reverse the calculation, or to omit the – sign from the answer.

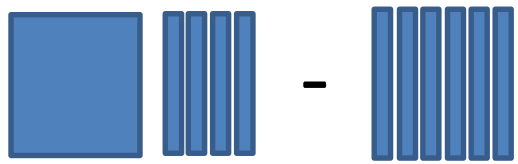
When finding the answer to the calculation, the children will need to have a secure understanding of the meaning of the + and – signs so that they can complete the calculation 200-20+1.

Formal Algorithms for…**Subtraction**

Stage 2:

300 140
~~400~~ + ~~40~~ + 8
\- 200 + 60 + 7

100 + 80 + 1 = 181



= 80

The understanding required for full decomposition can be developed by representing the ‘borrowing’ of 100 from the 400 with ‘Base 10’ resources. Adding the 100 to the 40 to make 140, means that you can now subtract 60 from 140 to equal 80.

This process means that you don’t end up with a negative number at the bottom and that the jump to decomposition is smaller.

This is due to retaining the understanding that when you ‘borrow’ you are ‘borrowing’ 10 or 100, not 1. This is consolidated by crossing out the 40, and writing 140 above it, rather than just tagging on a 1.

Stage 3:

3
~~4~~ 14 8
\- 2 6 7

1 8 1

In the example for **stage 3**, we have ‘borrowed’ the 100 from the hundreds column and added it to the tens column. Using ‘Base 10’ resources, you can highlight that the revised calculation shows the calculations: 8-7, **14 (tens) – 6 (tens),** 3 (hundreds) – 2 (hundreds). For understanding, the part to highlight is the *14 tens – 6 tens*, as this is where the misconception can be. (See diagram in **stage 2**)

Formal Algorithms for…**Multiplication**

When choosing the methods for calculating with multiplication, it is important to consider how the method helps you to understand the distributivity and commutativity of multiplication and whether the chosen method aids the calculation itself. This is the reason for placing an emphasis on the methods shown in the main body of the calculations policy.

When using the formal algorithms for multiplication, identifying the distributivity and commutativity is not always possible. For this reason, it is important that the children understand what they are calculating and why at each stage of the algorithm.

There are different methods for calculating multiplication (such as the 'Lattice method') which are not shown in this appendix. These are the methods that the children will need to know for the Key Stage 2 assessments.

Short Multiplication: ***342 x 7 =***

```
    3 4 2
x       7
---------
      1 4   (2x7)
    2 8 0   (40x7)
  2 1 0 0   (300x7)
---------
  2 3 9 4
```

This method highlights the distributive law of multiplication as, like the grid method, you have to separate the component parts of the number to perform the calculation. The misconceptions can occur when 40x7 becomes 4x7. The place value of the digits needs to be preserved.

```
    3 4 2
x       7
---------
  2 3 9 4
---------
    2 1
```

As addition, short multiplication should start in an expanded form, before progressing to the compact version.

Formal Algorithms for…**Multiplication**

146 x 84 =

Long Multiplication - Step 1:

	100	40	6
80	8000	3200	480
4	400	160	24

```
   1 4 6
 x   8 4
 -------
     2 4   (6x4)
   1 6 0   (40x4)
   4 0 0   (100x4)
   4 8 0   (6x80)
 3 2 0 0   (40x80)
 8 0 0 0   (100x80)
 -------
 1 2 2 6 4
 -------
```

For long multiplication, start with the conceptual understanding that is provided by the 'grid method' to scaffold the step towards the formal algorithm. Each method highlights the distributive law, especially when you start with the expanded form of long multiplication in **step 1**.

Long Multilication - Step 2:

```
     1 4 6
   x   8 4
 ---------
     5 8 4
     1 2
 1 1 6 8 0
   3 4
 ---------
 1 2 2 6 4
   1 1
```

As with short multiplication, the place value of each digit can be lost if 6x80 is read as 6x8. Misconceptions are formed when people say, 'just write a zero on the second line'. The reason for this is that we are multiplying by 80 rather than by 8. Children should be encouraged to say that the calculation is 6x80 and use their knowledge of the number fact 6x8 to help them find the product 480.

Formal Algorithms for…**Division**

As with multiplication, when choosing calculation methods for division, there are certain considerations to take into account. Does the method show that division is not commutative? Does it highlight that you can swap the divisor and the quotient, but you cannot swap the divisor and the dividend? Does the method aid calculation? Using arrays and the number line do highlight these issues, however with the bus stop method, it can be less clear.

Children should know how the bus stop methods for short and long division work, but are not expected to use them as their default method of calculation. To promote understanding of division and the calculation that is being performed, children should be encouraged to use the number line when dividing.

These methods should be learned so that children can answer questions in the Key Stage 2 assessments.

Short Division ***432 ÷ 5***

```
     8  6  r2
  5 | 4  3 ³2
```

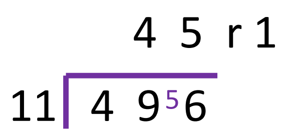
Answer = 86 r 2 or $86\frac{2}{5}$ or 86.4

432 ÷ 5 in isolation is a difficult calculation, mentally partitioning the number by place value in short division can help. You could calculate 400 ÷ 5 = 80, however this is unhelpful as recording the 80 would require writing the 8 in the tens column and ignoring the 0, which could be confusing. For this reason, calculate 43 ÷ 5, which gives 8, and then carry the 3 to make 32. This can then be divided by 5 to equal the 6 with 2 remainders.
The remainder can be left as it is, or divided by 5 to give ⅖ or 0.4.

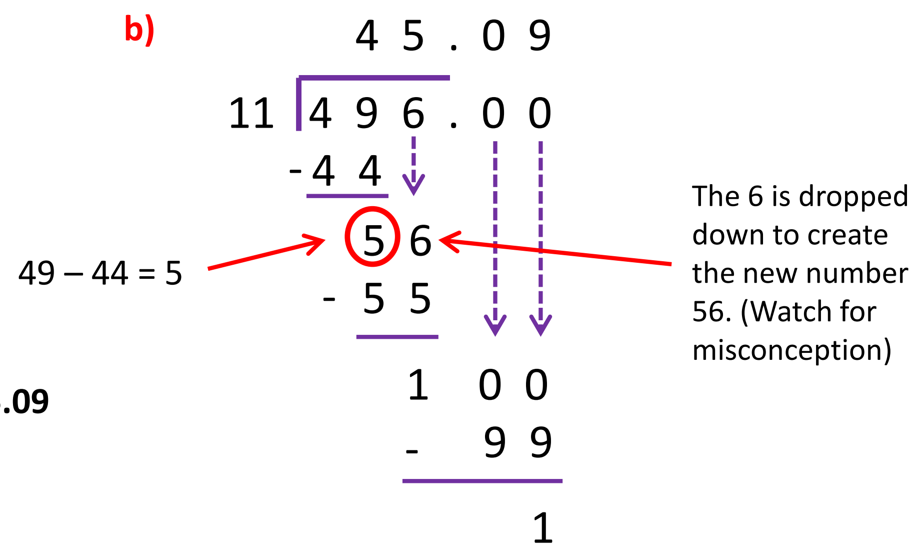
Formal Algorithms for…**Division**

Long Division ***496 ÷ 11***

a)



b)



Answer: 45 remainder 1 or 45 $\frac{1}{11}$ or 45.09

Long division is a method for dividing a 2 or 3 digit number by a number of at least 2 digits. Example **a)** shows that it is possible to use short division for this type of calculation to derive the answer 45r1 or 45$\frac{1}{11}$. To gain greater precision including decimal places, long division is required. Example **b)** shows how the first two digits of the dividend (496) is divided by 11 (49÷11) to equal 44 with 5 left over. This is shown as the subtraction beneath the dividend. You then drop the 6 down to make the 5 into 56. (Potential misconception here due to 5 becoming 5 tens). This is then divided by 11 and the remainder recorded as before. As the remainder is now 1, you drop a zero down to make 10, then once more to make it 100 (See previous place value misconception). This process can be continued to add additional decimal places as required.